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THESIS

A COMPUTER STUDY OF THE BUCKLING OF AN
IMPERFECT CYLINDRICAL SHELL

by

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A Computer Study of the Buckling of an Imperfect Cylindrical Shell

by

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ABSTRACT

An existing computer program for the geometrically non-linear analysis of arbitrarily loaded shells of revolution was modified to incorporate initial geometric imperfections into the buckling solution of an axially loaded circular cylindrical shell. Several different boundary conditions were considered, and the predicted buckling loads were examined to determine the significance of the initial imperfections and the boundary conditions for the shell stability problem. The computed buckling loads were compared to those found experimentally by Arbocz and Babcock at the California Institute of Technology. The computed results were generally higher than the experimental results. It was indicated that imperfection sensitivity is dependent on the boundary conditions of the cylinder.

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TABLE OF SYMBOLS

a	Reference length
b	Nondimensional inplane stiffness
d	Nondimensional bending stiffness
E	Young's modulus
E_o	Reference Young's modulus
h	Wall thickness
h_o	Reference thickness
L	Total length of meridian
m	Mode number in the meridional direction
M_s	Meridional bending moment
n	Mode number in the circumferential direction
N_o	Uniform axial load
$N_s, N_\theta, N_{s\theta}$	Membrane forces per unit length
Q_s	Transverse force per unit length
r	Radius
s	Meridional coordinate
$t_s, t_\theta, t_{s\theta}$	Nondimensional Fourier coefficients for the membrane forces
U	Displacement tangent to the meridian
V	Displacement tangent to the parallel circle
W	Displacement normal to the reference surface
$\bar{W}$	Displacement normal to the reference surface due to initial imperfections
β	Nondimensional coefficients for the nonlinear terms

$\varepsilon, \varepsilon_s, \varepsilon_{s\theta}$	Reference surface strains
η	Coordinate normal to the reference surface
θ	Circumferential angle
ν	Poisson's ratio
ξ	Nondimensional meridional coordinate
Φ	Reference surface rotation
σ_o	Reference stress
$\bar{\phi}$	Nondimensional coefficients for surface rotations due to initial imperfections

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I. INTRODUCTION

The study of the instability of thin, isotropic, circular cylindrical shells has long been troubled with a lack of correlation between the theoretically predicted buckling loads and the experimental results; the experimental values of buckling are generally much lower than the theoretical solutions. This lack of agreement has been attributed to imperfection sensitivity, i.e., inherent imperfections in the shape of the shell which are unaccounted for in the classical buckling theory trigger the buckling modes prematurely, and to the use of incorrect boundary conditions in the analysis, i.e., the boundary conditions used in the analysis are not equivalent to those of the test specimen [1].

The recent development of several sophisticated digital computer programs for the geometrically nonlinear analysis of shell structures has made it possible to include both the initial imperfections and arbitrary boundary conditions in the analysis. For example, a digital computer program for the geometrical nonlinear analysis of arbitrarily loaded shells of revolution has been developed by Ball [2]. A modification has recently been made to this program to include the initial imperfections in the analysis. However, this modification does not eliminate all of the previous analytical difficulties because in order to predict the buckling load of a particular shell, the imperfect shape of

that shell must be known, and the actual boundary conditions of the specimen must be known. With regard to the first problem, Arbocz and Babcock [3] have developed a means of mapping the topography of cylindrical shells and can thus describe the deviations from a perfect circular cylinder. The objective of this thesis is to use the imperfection data for one of the cylinders presented in Ref. [3] and Ball's modified computer program to obtain stability solutions for several boundary conditions. The buckling loads are compared with the experimental results presented in Ref. [3] and are compared with each other to determine the effect of the various boundary conditions.

II. DESCRIPTION OF THE COMPUTER PROGRAM

The computer program used in this study is capable of analyzing thin isotropic shells under the following conditions:

1) The geometric and material properties must be axisymmetric, but may vary along a meridian.

2) The initial imperfections, pressures, and temperatures must be symmetric about a meridional plane.

3) The boundaries may be free, fixed, or elastically restrained.

The governing partial differential equations are based on Sanders' nonlinear shell theory for the condition of small strains and moderately small rotations (Appendix A). Inplane and normal forces are accounted for but rotary inertial forces are neglected. The set of partial differential equations is reduced to an infinite number of sets of four second-order differential equations in the meridional and time coordinates by expansion of the dependent variables in a sine or cosine series in the argument $n\theta$ where θ is the circumferential coordinate and n is the Fourier index. Nonlinear coupling terms are treated as pseudo loads and trigonometric identities are used to uncouple the sets of equations. Derivatives with respect to the meridional coordinates are replaced by the conventional finite difference approximation. This leads to sets of algebraic

equations in the four dependent variables $U^{(n)}$, $V^{(n)}$, $W^{(n)}$, and $M_s^{(n)}$ where U, V, and W are the displacements in the meridional, circumferential, and normal directions respectively, and M_s is the meridional bending moment per unit length. At each load or time step, an estimate of the solution is obtained by extropolation from solutions at the previous steps. The sets of equations are repeatedly solved using Gaussian elimination, and the pseudo loads are recomputed until the solution converges. If convergence is not achieved in a specified number of iterations, the load is reduced by a factor of five. If, after a specified number of load reductions, the solution fails to converge the problem is terminated. If the load-displacement behavior of the shell is of the softening type, buckling is presumed to have occurred at the final load.

The original program described in [2] was not designed to solve shell stability problems where initial imperfections had to be accounted for. However, it soon became apparent that such a capability was very desirable and the program was modified accordingly. The equations and FORTRAN statements that include the imperfection terms are presented in Appendix A.

III. BOUNDARY CONDITIONS

In the analysis, the boundary conditions for the cylinder are on N_s or U, $N_{s\theta}$ or V, Q_s or W, and $\partial W/\partial s$ or M_s, where N_s and $N_{s\theta}$ are membrane forces per unit length, and Q_s is the transverse force per unit length. The boundary conditions are represented by the matrix equation

$$[\Omega]\begin{Bmatrix} N_s \\ N_{s\theta} \\ Q_s \\ \dfrac{\partial W}{\partial s} \end{Bmatrix} + [\Lambda]\begin{Bmatrix} U \\ V \\ W \\ M_s \end{Bmatrix} = \{\ell\}$$

where $[\Omega]$ and $[\Lambda]$ are 4 x 4 matrices and $\{\ell\}$ is a 1 x 4 column matrix. There is one such equation for each boundary of the shell. Note that the analysis assumes that the boundary conditions are the same for all values of n, where n is the mode number in the circumferential direction.

The testing configuration used in Ref. [3] for the axially loaded shell is shown in Fig. 1. The ends of the shell are attached to an end ring and a load cell with Cerrelow, a low melting point alloy. The ends of the shell are clamped such that

$$W = \partial W/\partial s = V = 0 \tag{1}$$

The axial load on the test specimen was applied by means of four screws located at the corners of the square end

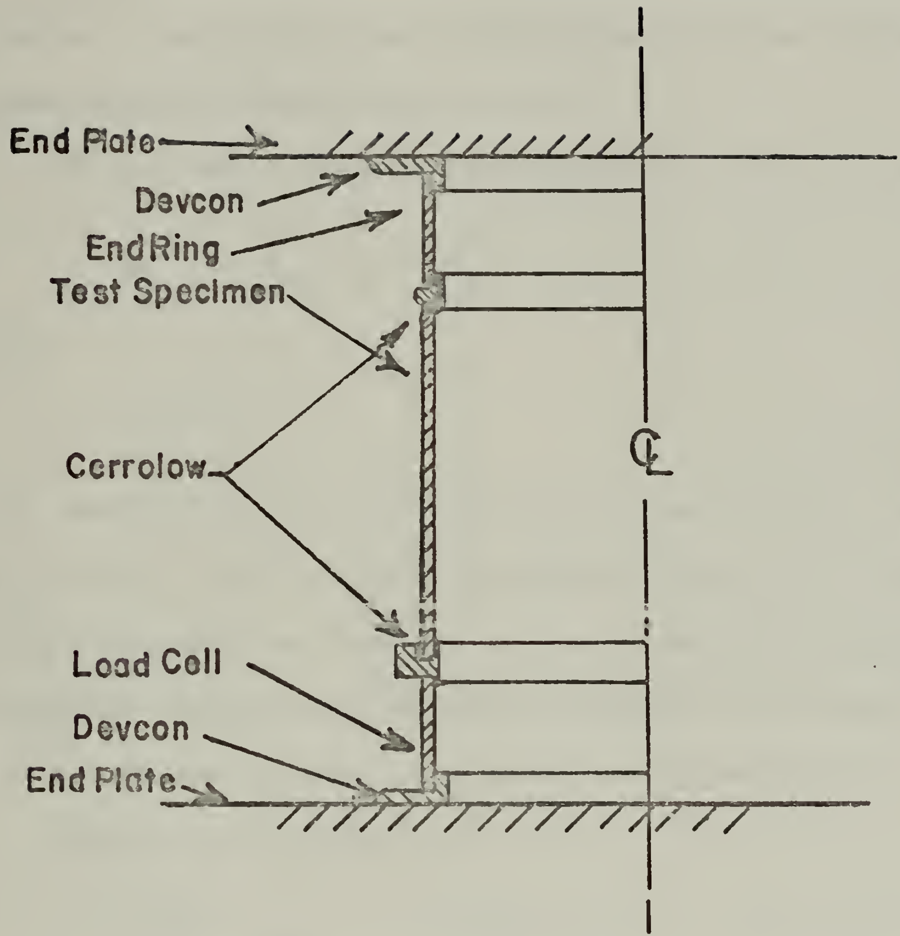

FIG. I CYLINDRICAL SHELL TESTING CONFIGURATION

plates. At each load increment, the screws were adjusted to give as uniform a circumferential load distribution as possible.

In order to investigate the problem completely, several boundary conditions were formulated for the computer program. These formulations follow below.

If the axial load is axisymmetric, then on each boundary

$$N_s^{(0)} = N_o \quad , \quad N_s^{(n)} = 0 \qquad n = 1,2,...$$

and (2)

$$U^{(n)} \neq 0 \qquad n = 0,1,2,...$$

This condition will be referred to as boundary condition A. Note in Eq. (2) that the condition on N_s is a function of n. This necessitates changes in the program to account for different values of the first element of ℓ for n = 0 and n ≠ 0. These changes are given in Appendix B.

Another possibility is

$$N_s^{(0)} = N_o \quad , \quad N_s^{(n)} = 0 \qquad n = 1,2,...$$

at one end of the cylinder, and

$$U^{(n)} = 0 \quad , \quad N_s^{(n)} \neq 0 \qquad n = 0,1,2,...$$

at the other end. This condition will be referred to as boundary condition B.

A third interpretation of the test set-up consists of rigid end plates incrementally shortening the cylinder. This is represented at the boundaries by

$$U^{(0)} = U_o \quad , \quad U^{(n)} = 0 \qquad n = 1,2,\ldots$$

at one end, and

$$U^{(n)} = 0 \qquad n = 0,1,2,\ldots$$

at the other end. This condition will be referred to as boundary condition C.

For purposes of comparison, a simply supported cylinder will also be analyzed using boundary conditions A, B, and C. For this case, equation (1) becomes

$$W = M_s = V = 0 \tag{3}$$

IV. INITIAL IMPERFECTIONS

Arbocz and Babcock's topography mapping procedure is described in detail in Ref. [3]. The final imperfection data consisted of deflections from a perfectly circular cylinder, denoted here by $\overline{W}$, around fifteen circumferences spaced 0.5 in. apart. The first and the last circumferential mappings occurred at a distance of 0.125 in. from the ends of the shell. Forty-nine data points were taken around each circumference with the first and the last being the same point on one arbitrarily chosen meridian.

As noted previously, Ball's program requires a meridional plane about which the imperfect cylinder is symmetric. This requirement is not generally met by imperfect cylinders; thus symmetry must be created in some optimum way. An investigation to determine the degree of symmetry was conducted by generating a cosine Fourier series for each circumference using every meridian of data as a base meridian. In all cases, the phase angles for each circumference varied considerably, thus eliminating the possibility that the imperfections were symmetric. Consequently, a least squares method was used to determine the best meridian of data points about which to create symmetry. This best meridian was used as the origin to form the series

$$\overline{W} = \sum_{n=0}^{23} \overline{W}^{(n)} \cos(n\theta)$$

using the data points from $\theta = 0$ to $\theta = \pi$.

An attempt was made to create a finer meridional mesh of the imperfection data by passing a general polynomial through the fifteen known coefficients for each value of n in order to interpolate imperfection coefficients at intermediate stations. However, a polynomial of sufficient degree to pass through all of the stations resulted in unreliable coefficients at intermediate stations. Conversely, a least squares fit using a lower order polynomial resulted in a reasonable coefficient representation, but the coefficients at the original fifteen stations were compromised to an extent that depended on the degree of the polynomial. Similar problems were encountered using a Fourier series representation. As a result of the difficulty in establishing coefficients at intermediate stations, the fifteen original locations were selected as the stations for the numerical shell analysis and the conventional central finite difference scheme was used to obtain the slope of the imperfection shape away from the boundaries. At the boundaries the simplest forward and backward differencing schemes were used.

V. AN EIGENVALUE SOLUTION

To provide a check on the validity of the solution from the computer program, buckling of a simply supported perfectly circular cylindrical shell was formulated as an eigenvalue problem and solved analytically. The set of uncoupled field equations can be given in the form

$$[E^{(n)}]\ \{Z^{(n)''}\} + [F^{(n)}]\ \{Z^{(n)'}\} + [G^{(n)}]\ \{Z^{(n)}\} = \{e^{(n)}\} \quad (4)$$

where

$$Z = \{U^{(n)},\ V^{(n)},\ W^{(n)},\ M_s^{(n)}\}^T .$$

The equations for the values of the elements in the $[E^{(n)}]$, $[F^{(n)}]$, $[G^{(n)}]$, and $\{e^{(n)}\}$ matrices are given in Appendix C.

For n = 0, the linear membrane solution $N_s^{(0)} = -N_o$ is assumed. Thus the buckling modes of the shell for the boundary conditions given in equations (2) (boundary condition A) and (3) are

$$\begin{aligned} U^{(n)} &= C_1 \cos m\pi s/L \\ V^{(n)} &= C_2 \sin m\pi s/L \\ W^{(n)} &= C_3 \sin m\pi s/L \\ M_s^{(n)} &= C_4 \sin m\pi s/L \end{aligned} \quad (5)$$

where $n \neq 0$, $C_1 - C_4$ are arbitrary constants, m denotes the axial mode number and L is the length of the cylinder.

Substituting equations (5) into equations (4) results in an eigenvalue problem of the form

$$[A]\ \{C\} = -N_o[B]\{C\}$$

where [A] and [B] are 4 x 4 matrices defined in Appendix C and

$$\{C\} = \{C_1,C_2,C_3,C_4\}^T.$$

Because B is singular, the inverse formulation

$$\frac{1}{N_o}\ [A]\{C\} = -[B]\{C\} \tag{6}$$

is used.

VI. SHELL DESCRIPTION AND INPUT DATA

The shell used in this study was cylinder A-8 of Ref. 3 Its dimensions and properties are:

Radius, r, R_θ	= 4.0 in.
Nominal length	= 8.25 in.
Wall thickness, h	= 0.00464 in.
Young's modulus, E	= 15.2×10^6 lb/in.
Poisson's ratio, ν	= 0.333

Six harmonics of the initial imperfection data were used as input to the program. They are n = 0, 9, 10, 11, 12, and 13. The selection of these particular harmonics was based on the eigenvalue solution for the simply supported shell that predicated a minimum buckling load when m = 1 and n = 9, and the experimental observation in Ref. [3] that n = 13 appeared to be the critical mode where m and n are the mode shapes in the meridional and circumferential directions respectively. The number of meridional stations used was fifteen because no reliable interpolation of the initial imperfections could be developed, as discussed above. The length of the shell, L, was taken as 8 in. with the first and the last data point stations assumed to be the stations at the edges of the shell. Convergence criterion was taken as 0.01.

As presented in Appendix A, the imperfection input data consists of the rotations $\bar{\phi}_{s_k}^{(n)}$ and $\bar{\phi}_{\theta_k}^{(n)}$ where

$$\bar{\phi}_{s_k}^{(n)} = -\frac{\sigma_o}{E_o}\left(\frac{dW}{ds}^{(n)}\right)_k \quad \text{and} \quad \bar{\phi}_{\theta_k}^{(n)} = \frac{a\sigma_o}{rE_o} nW_k^{(n)}$$

in which σ_o is a reference stress level, and E_o is a reference elastic modulus, a is a reference length, and k denotes the station number. The data are introduced into the program through the subroutine IMPERF. The FORTRAN statements for IMPERF are given in Appendix D. The imperfection data was provided to the author by the writers of Ref. [3] in the form of a deck of cards with the data $\overline{W}/h$.

In order to obtain a buckling load that was a very close approximation to the bifurcation load of the eigenvalue problem, the imperfections were reduced by a factor 10^{-6}. This technique has been used successfully with this program by Stillwell [4] who used very small asymmetries in the load as the triggering mechanism.

VII. RESULTS AND DISCUSSION

The solution to the eigenvalue problem given by equation (6) resulted in a minimum buckling load for the simply supported cylinder of 53.02 lb/in when $m = 1$ and $n = 9$. The solution from the computer program for the minute imperfections resulted in a buckling load of 54.0 lb/in for boundary condition A, simply supported edges, and displacement in harmonics $n = 0$ and $n = 9$. Some difference between the two results is to be expected because the analytical solution neglects the pre-buckling deformation due to edge constraint, and the numerical solution used only fifteen stations.

The computer results for the simply supported cylinder with boundary conditions A, B, and C, with both minute and full imperfections,* are presented in Table I. These results show a relationship between the imperfection sensitivity of the shell and the boundary conditions. For example, boundary condition A shows a 15% reduction in the load carrying capacity when the full imperfection is introduced. Boundary condition B shows a 10% reduction, while condition C reduces the capacity by 42.7%.

The computer results for the clamped shell are given in Table II. The experimental buckling load of shell A-8 was

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*minute = full imperfections x 10^{-6}.

TABLE I

SIMPLY SUPPORTED CYLINDER

Loading Condition	Degree of Imperfection	Buckling Load (lb/in)
A	minute	54.0
A	full	45.9
B	minute	69.0
B	full	62.7
C	minute	78.6
C	full	45.0

TABLE II

CLAMPED CYLINDER

Loading Condition	Degree of Imperfection	Buckling Load (lb/in)
A	minute	7.5
A	full	no convergence
B	minute	43.87
B	full	46.9
C	minute	85.7
C	full	49.1

found to be 32.87 lb/in [3]. From Table II:

1) The buckling load for condition A with minute imperfections is 7.5 lb/in.*

2) No converged solution could be found for condition A with full imperfections.

3) The buckling load for condition B is slightly higher with full imperfections than for the minute case.

4) The buckling load for loading condition C is decreased by 42.8% when the full imperfections are introduced.

Thus, the stability of the clamped shell under condition A is drastically reduced below that of condition B and C. This is very puzzling and much effort was expended in search of an explanation. Unfortunately, none was found. The computed result for condition C, although generally higher than the empirical result, shows the expected load reduction due to the initial imperfections [3]. Some of the discrepancy in magnitude could be due to the use of an effective length that was 0.25 in. less than the actual length of the shell since the greater length would lower the computed buckling load. The remainder may be due to the fact that the actual boundary condition is some combination of conditions A and C. Another consideration is that the symmetric representation of the actual imperfections may also degrade the validity of the solution.

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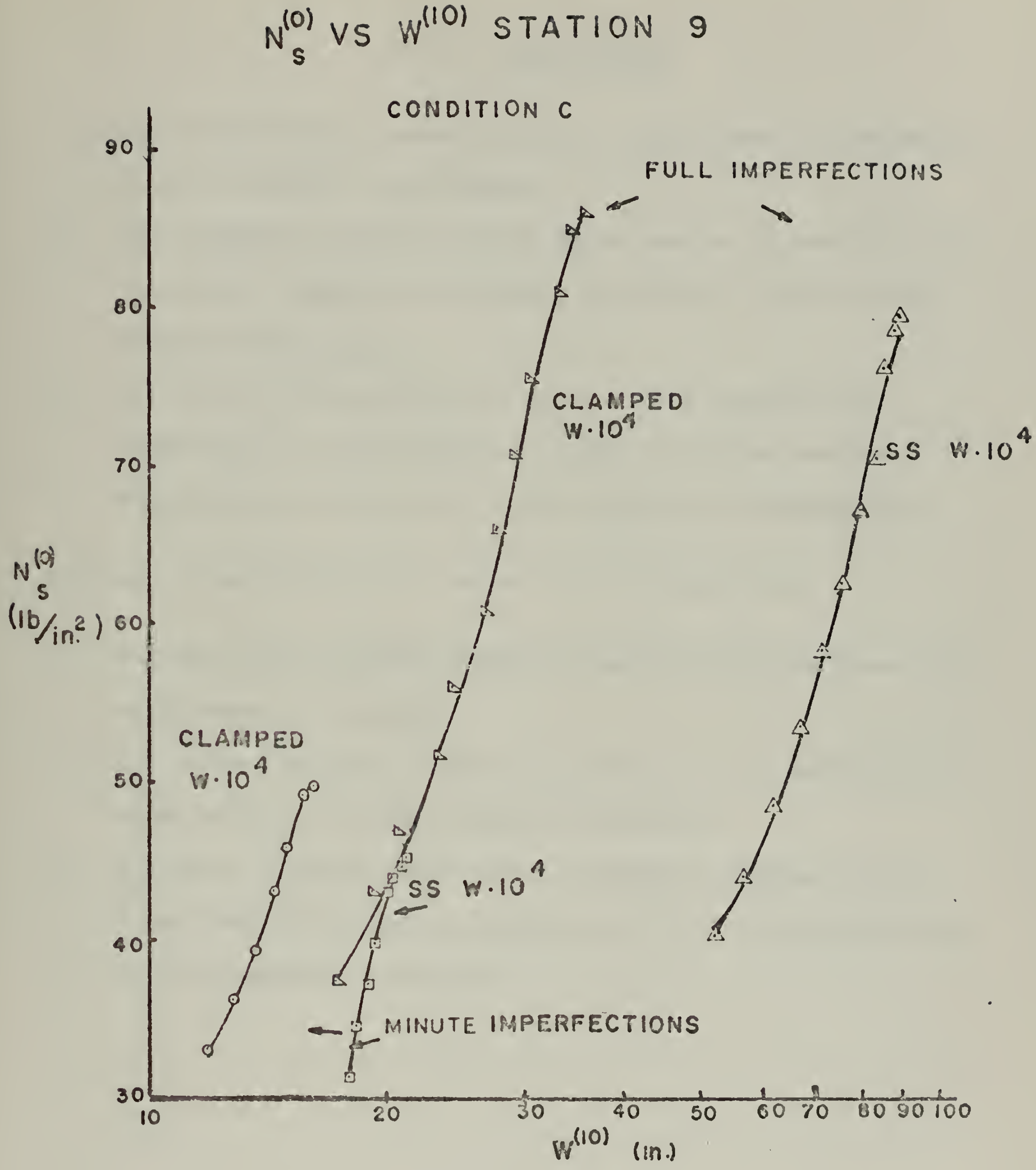
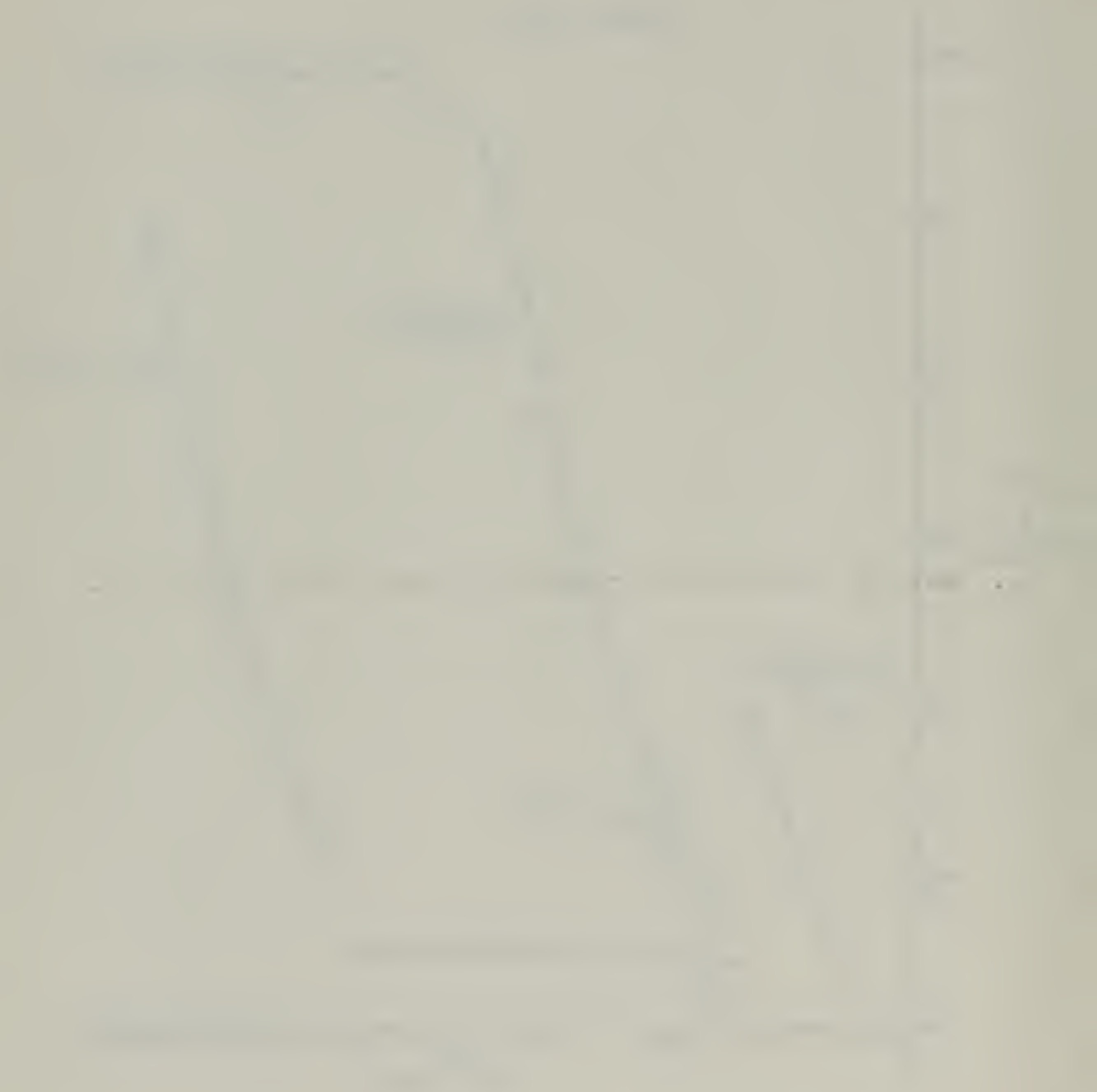
*For this boundary condition, the imperfections were reduced by a factor of 10^{-9}.

The circumferential harmonic $n = 10$ was found to exhibit the most rapid growth. Figure 2 displays $N_s^{(0)}$ vs $W^{(10)}$ for boundary condition C. The sudden decrease in slope occurring just prior to reaching $N_{crit.}^{(0)}$ illustrates the buckling (non-convergence) phenomenon.



N_s^(0) VS W^(10) STATION 9
CONDITION C
FULL IMPERFECTIONS
CLAMPED W·10^4
SS W·10^4
CLAMPED W·10^4
SS W·10^4
MINUTE IMPERFECTIONS
N_s^(0) (lb/in.^2)
90
80
70
60
50
40
30
10
20
30
40
50
60
70
80
90
100
W^(10) (in.)

FIG.2



VIII. CONCLUSIONS

1. The imperfection sensitivity of a cylinder is dependent on the boundary conditions.
2. The computer program yields more realistic results for the simply supported boundary condition than for the clamped condition.
3. The initial imperfections result in a significant reduction in load carrying capacity in the simply supported case but less reduction than expected [3].

Further investigation is necessary to determine:

1. Why are the computed results generally higher than the experimental results?
2. Why is the clamped boundary condition A so sensitive to imperfections in the computer program?
3. Why does loading condition B, clamped, appear to be strengthened by the introduction of full imperfections in the computer solution?

APPENDIX A

This section presents the appropriate equations for including initial imperfections in the computer program. The strain-displacement relationships used in this analysis are

$$\varepsilon_s = U' + W/R_s + (\Phi_s^2 + \Phi^2)/2$$

$$\varepsilon_\theta = V^{\cdot}/r + r'U/R + W/R + (\Phi^2 + \Phi^2)/2 \qquad \text{(A1)}$$

$$\varepsilon_{s\theta} = (V' + U^{\cdot}/r - r'V/r + \Phi_s\Phi_\theta)/2$$

where

$$\Phi_s = -W' + U/R_s$$

$$\Phi_\theta = -W^{\cdot}/r + V/R_\theta \qquad \text{(A2)}$$

$$\Phi = (V' + r'V/r - U^{\cdot}/r)/2$$

and where ε_s, ε_θ, $\varepsilon_{s\theta}$ represent Fourier coefficients of the reference surface strains, Φ, Φ_θ, and $\Phi_{s\theta}$ represent reference surface rotations, and R_θ and R_s are the principal radii of curvature. The superscript primes and dots denote partial differentiation with respect to s and θ respectively.

Consider an imperfect cylinder. Before loading, the imperfections appear as a displacement from the perfect cylinder. Note that this is a displacement with no accompanying strain. Call this displacement $\overline{W}$, and the total displacement from the perfect shape W*. Then

$$W^* = W + \overline{W}$$

where W is that part of the total displacement caused by the applied load and is associated with strain. Replacing W with W* in equations (A1) and (A2) and requiring that $\varepsilon_s = \varepsilon_\theta = \varepsilon_{s\theta} = 0$ when $U = V = W = 0$, leads to

$$\varepsilon_s = U' + W/R_s + [(-W'+U/R_s)^2 - 2\overline{W}'(-W^\cdot+U/R_s) + \Phi^2]/2$$

$$\varepsilon_\theta = V^\cdot/r + r'U/r + W/R_\theta + [(-W^\cdot/r + V/R\ )^2 - 2\overline{W}^\cdot/r(-W^\cdot/r+V/R_\theta) + \Phi^2]/2$$

$$\varepsilon_{s\theta} = [V'+U^\cdot/r - r'V/r + (-W'+U/R_s)(-W^\cdot/r+V/R_\theta) - \overline{W}'(-W^\cdot/r + V/R_\theta) - \overline{W}^\cdot/r(-W' + U/R_s)]/2$$

Defining the imperfection rotations $\overline{\Phi}_s$ and $\overline{\Phi}_\theta$ as

$$\overline{\Phi}_s = -\overline{W}' \qquad \text{and} \qquad \overline{\Phi}_\theta = -\overline{W}^\cdot/r$$

equations (A2) become

$$\varepsilon_s = U'+W/R_s + (\Phi_s^2 + 2\overline{\Phi}_s\Phi_s + \Phi^2)/2$$

$$\varepsilon_\theta = V^\cdot/r + r'U/r + W/R_\theta + (\Phi_\theta^2 + 2\overline{\Phi}_\theta\Phi_\theta + \Phi^2)/2$$

$$\varepsilon_{s\theta} = (V'+U^\cdot/r - r'V/r + \Phi_s\Phi_\theta + \overline{\Phi}_s\overline{\Phi}_\theta + \Phi_s\overline{\Phi}_\theta)/2$$

From Ref. [2]

$$\phi_s^2 = \left(\frac{\sigma_o}{E_o}\right)^2 \left(\sum_{n=0}^{\infty} \phi_s^{(n)} \cos n\theta\right)^2 = \frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} \beta_s^{(n)} \cos n\theta$$

Redefine β_s such that

$$\frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} \beta_s^{(n)} \cos n\theta = \Phi_s^2 + 2\overline{\Phi}_s \Phi_s$$

$$= \frac{\sigma_o}{E_o} \left\{ \left(\sum_{n=0}^{\infty} \phi_s^{(n)} \cos n\theta \right)^2 + 2 \left(\sum_{n=0}^{\infty} \overline{\phi}_s^{(n)} \cos n\theta \right) \left(\sum_{n=0}^{\infty} \phi_s^{(n)} \cos n\theta \right) \right\}$$

Similarly

$$\frac{\sigma_o}{E_o} \sum_{n=0}^{\infty} \beta_\theta \cos n\theta = \Phi_\theta^2 + 2\,\overline{\Phi}_\theta \Phi_\theta$$

$$= \frac{\sigma_o}{E_o} \left\{ \left(\sum_{n=1}^{\infty} \phi_\theta^{n} \cos n\theta \right)^2 + 2 \left(\sum_{n=1}^{\infty} \overline{\phi}_\theta^{(n)} \sin n\theta \right) \left(\sum_{n=1}^{\infty} \phi_\theta^{(n)} \sin n\theta \right) \right\}$$

and

$$\frac{\sigma_o}{E_o} \sum_{n=1}^{\infty} \beta_{s\theta} \sin n\theta = \Phi_s \Phi_\theta + \overline{\Phi}_s \Phi_\theta + \Phi_s \overline{\Phi}_\theta$$

$$= \left(\frac{\sigma_o}{E_o} \right)^2 \left\{ \left(\sum_{n=0}^{\infty} \phi_s^{(n)} \cos n\theta \right) \left(\sum_{n=1}^{\infty} \phi_\theta^{(n)} \sin n\theta \right) + \right.$$

$$\left(\sum_{n=0}^{\infty} \overline{\phi}_s^{(n)} \cos n\theta \right) \left(\sum_{n=1}^{\infty} \phi_\theta^{(n)} \sin n\theta \right) + \left(\sum_{n=0}^{\infty} \phi_s^{(n)} \cos n\theta \right)$$

$$\left. \left(\sum_{n=1}^{\infty} \overline{\phi}_\theta^{(n)} \sin n\theta \right) \right\}$$

where

$$\overline{\phi}_s^{(n)} = - \frac{d\overline{w}^{(n)}}{d\xi}$$

and

$$\overline{\phi}_\theta^{(n)} = \frac{n\overline{w}^{(n)}}{\rho}$$

and $\xi = s/a$, $\rho = r/a$, and $\overline{w}^{(n)} = E_o \overline{W}^{(n)} / (a\sigma_o)$.

The changes to the program are in SUBROUTINE PHIBET given below.

```
      SUBROUTINE PHIBET(K)
      COMMON
     1/IBL1/MNMAX
     2/IBL2/N(10),MNINIT
     3/IBL4/KMAX,KL
     3/IBL7/MNMAXO,MAXD(10),MAXS(10),MAXSY(10),IS(10,10),JS(10,10),ID(10
     4,10),JD(10,10),IJS(10)
     6/IBL12/KMAX1,KMAX2,NCONV
     2/IBL13/ITRMAX,LSMAX
     8/BL6/Z(4,220),SOE,OSE,ALOAD
     0/BL8/R(200),GAM(200),OMT(200)
     9/BL10/PHIX(10),PHIT(10),PHI(10)
     3/BL11/OMXI(200),PHEE,TO,T2
     1/BL12/TOLI,TDEL
     2/BL15/NU,U1(10),V1(10),W1(10),V2(10),U2(10),W2(10),U3(10),V3(10),
     3W3(10)
     4/BL27/BX3(10),BT3(10),BXT3(10),BE3(10)
      COMMON/BLIMP/PHIXB(200),PHITB(200)
      COMMON/BLPHS/PHX(10),PHT(10)
      OX=OMXI(K)
      OT=OMT(K)
      RRA=1./R(K)
      GA=GAM(K)
      KP2=K+2
      DO 1 M=1,MNMAXO
      EN=N(M)
      IK=KP2+(M-1)*KMAX2
      U3(M)=Z(1,IK)
      V3(M)=Z(2,IK)
      W3(M)=Z(3,IK)
      PHIX(M)=-TOLI*(W3(M)-W1(M))+OX*U2(M)
      PHIT(M)=EN*W2(M)*RRA+V2(M)*OT
    1 PHI(M)=(TOLI*(V3(M)-V1(M))+GA*V2(M)+EN*U2(M)*RRA)*.5
      IF(ITRMAX.EQ.1) RETURN
              DO 60 M=1,MNMAXO
      KP=K+(M-1)*KMAX
      PHX(M)=PHIXB(KP)
   60 PHT(M)=PHITB(KP)
      DO 9 M=1,MNMAX
      SMO=0.
      SMT=0.
      SMR=0.
      SMF=0.
      IF(N(M).EQ.0) GO TO 20
      MAXL=MAXS(M)
      IF(MAXL.EQ.0) GO TO 2
      DO 3 L=1,MAXL
      I=IS(L,M)
      J=JS(L,M)
      SMO=SMO+PHIX(I)*PHIX(J)
     1     +    PHX(I)*PHIX(J)   +PHX(J)*PHIX(I)
      SMT=SMT-PHIT(I)*PHIT(J)
     1     -    PHT(I)*PHIT(J)    -PHT(J)*PHIT(I)
      SMR=SMR+PHIX(I)*PHIT(J)+PHIX(J)*PHIT(I)
     1    +PHX(I)*PHIT(J)+PHX(J)*PHIT(I)+PHIX(I)*PHT(J)+PHIX(J)*PHT(I)
    3 SMF=SMF-PHI(I)*PHI(J)
    2 MAXL=MAXD(M)
      IF(MAXL.EQ.0) GO TO 4
      DO 5 L=1,MAXL
      I=ID(L,M)
      J=JD(L,M)
      SMO=SMO+PHIX(I)*PHIX(J)
     1     +    PHX(I)*PHIX(J)   +PHX(J)*PHIX(I)
      SMT=SMT+PHIT(I)*PHIT(J)
     1     +   PHT(I)*PHIT(J)      +PHT(J)*PHIT(I)
      SMR=SMR-PHIX(I)*PHIT(J)+PHIX(J)*PHIT(I)
     1   -PHX(I)*PHIT(J)+PHX(J)*PHIT(I)-PHIX(I)*PHT(J)+PHIX(J)*PHT(I)
    5 SMF=SMF+PHI(I)*PHI(J)
    4 IF(MAXSY(M).EQ.0) GO TO 10
      I=IJS(M)
      SMO=SMO+PHIX(I)**2/2.
```

```
1     +PHX(I)*PHIX(I)
 SMT=SMT-PHIT(I)**2/2.
1    -PHT(I)*PHIT(I)
 SMR=(SMR+PHIX(I)*PHIT(I))
1     +PHX(I)*PHIT(I)+PHIX(I)*PHT(I)
 SMF=SMF-PHI(I)**2/2.
 GO TO 10
20 DO 21 L=1,MNMAXO
 SMO=SMO+PHIX(L)**2
1    +2.*PHX(L)*PHIX(L)
 SMT=SMT+PHIT(L)**2
1    +2.*PHT(L)*PHIT(L)
21 SMF=SMF+PHI(L)**2
 IF(M.GT.MNMAXO) GO TO 11
 SMO=SMO+PHIX(M)**2
1    +2.*PHX(M)*PHIX(M)
11 BX3(M)=SMO*.5
 BT3(M)=SMT*.5
 BE3(M)=SMF*.5
 BXT3(M)=0.
 GO TO 9
10 BX3(M)=SMO
 BT3(M)=SMT
 BXT3(M)=SMR*.5
 BE3(M)=SMF
9 CONTINUE
 RETURN
 END
```

APPENDIX B

The load applied in the computer program is uniform and taken in the first mode only, i.e., $n = 1$. To accomplish this, the following changes were made to the program logic:

```
      SUBROUTINE ZANDZ                           531

      DO 15 J = 1, 4                             659
*     IF(M.NE.1) ELLS(J) = 0

      RETURN
      END
```

```
      SUBROUTINE FORCE(K)                        960

      DO 21 J = 1, 4                             081
*     IF(M.NE.1) ELLS(J) = 0

      RETURN
      END
```

---

* = Required inserts

APPENDIX C

The elements of the [A] and [B] matrices used in the eigenvalue solution are defined below.

$A_{11} = G_{11} - E_{11}Q^2$

$A_{12} = F_{12}Q$

$A_{13} = F_{13}Q$

$A_{14} = 0$

$A_{21} = -F_{21}Q$

$A_{22} = -E_{22}Q^2 + G_{22}$

$A_{23} = G_{23} + E_{23}Q^2$

$A_{24} = G_{24}$

$A_{31} = -F_{31}Q$

$A_{32} = G_{32} - E_{32}Q^2$

$A_{33} = G_{33} - E_{33}Q^2$

$A_{34} = G_{34} - E_{34}Q^2$

$A_{41} = 0$

$A_{42} = G_{42}$

$A_{43} = G_{43} - E_{43}Q^2$

$A_{44} = G_{44}$

$B_{11} = 1/4\ n^2$

$B_{12} = 1/4\ nQ$

$B_{13} = 0$

$B_{14} = 0$

$B_{21} = 1/4\ nQ$

$B_{22} = 1/4\ Q^2$

$B_{23} = 0$

$B_{24} = 0$

$B_{31} = 0$

$B_{32} = 0$

$B_{33} = Q^2$

$B_{34} = 0$

$B_{41} = 0$

$B_{42} = 0$

$B_{43} = 0$

$B_{44} = 0$

where $Q = n\pi s/L$

and

$$E_{11} = b \qquad E_{21} = 0$$

$$E_{12} = 0 \qquad E_{22} = \frac{b(1-\nu)}{2} + \frac{d(1-\nu)}{8}\left(\frac{3}{R_\theta}\right)^2$$

$$E_{13} = 0 \qquad E_{23} = \frac{(1-\nu)}{2r}\ \frac{3n}{R_\theta}$$

$$E_{14} = 0 \qquad E_{24} = 0$$

$$E_{31} = 0 \qquad E_{41} = 0$$

$$E_{32} = E_{23} \qquad E_{42} = 0$$

$$E_{33} = \frac{d(1-\nu)}{r^2}\ 2n^2 \qquad E_{43} = -d$$

$$E_{34} = 1 \qquad E_{44} = 0$$

and

$$F_{11} = 0 \qquad F_{21} = -F_{12}$$

$$F_{12} = \frac{(1+\nu)bn}{2r}\ \frac{+dn(1-\nu)}{8r}\left(\frac{-3}{R_\theta{}^2}\right) \qquad F_{22} = 0$$

$$F_{13} = b\nu - d\ \frac{(1-\nu)n^2}{2r^2 R_\theta} \qquad F_{23} = 0$$

$$F_{14} = 0 \qquad F_{24} = 0$$

$$F_{31} = -F_{13} \qquad F_{41} = 0$$

$$F_{32} = 0 \qquad F_{42} = 0$$

$$F_{33} = 0 \qquad F_{43} = 0$$

$$F_{34} = 0 \qquad F_{44} = 0$$

and

$$G_{11} = -\left(\frac{(1-\nu)bn^2}{2r^2} + d(1-\nu)\left(\frac{-1}{8R_\theta r^2}\right)n^2\right)$$

$$G_{12} = 0$$

$$G_{13} = 0$$

$$G_{14} = 0$$

$$G_{21} = 0$$

$$G_{22} = 0$$

$$G_{23} = \frac{-bn}{rR_\theta} + \left(\frac{dn(1-\nu)}{2r}\right)\left(\frac{-2(1+\nu)n^2}{r^2 R_\theta}\right)$$

$$G_{24} = \frac{-\nu n}{r}$$

$$G_{31} = 0$$

$$G_{32} = \frac{-bn}{rR_\theta} + \frac{d(1-\nu)n}{2r}\ (2(1+\nu)\left(\frac{-n^2}{r^2 R_\theta}\right)$$

$$G_{33} = \frac{-b}{R_\theta^{\ 2}} + \frac{d(1-\nu)n^2}{r^2}\ (1+\nu)\left(\frac{-n^2}{r^2}\right)$$

$$G_{34} = -\left(\frac{\nu n^2}{r^2}\right)$$

$$G_{41} = 0$$

$$G_{42} = \frac{d\nu n}{rR_\theta}$$

$$G_{43} = \frac{d\nu n^2}{r^2}$$

$$G_{44} = 1$$

APPENDIX D

The following subroutine was used to calculate $\bar{\phi}_{s_k}^{(n)}$ and $\bar{\phi}_{\theta_k}^{(n)}$ in the computer program.

```
      SUBROUTINE IMPERF
      DIMENSION FNT(15),DFNT(15)
      COMMON /BLIMP/PHITB(200),PHIXB(200)
      AO=4
      SIGA=1000.
      EO=15200000.
      H=8./14.
      H1=1./H
      H2=1./(2.*H)
      DO 1 I=1,6
      AJ2=I+7
      IF(I.NE.1) GO TO 8
      AJ2=0.
    8 READ(5,10) (FNT(J),J=1,15)
      DO 2 K=2,14
    2 DFNT(K)=(FNT(K+1)-FNT(K-1))*H2
      DFNT(1)=(FNT(2)-FNT(1)*H1
      DFNT(15)=(FNT(15)-FNT(14))*H1
      KR=I-1
      DO 3 K1=1,15
      KZ=KR*15+K1
      PHITB(KZ)=(FNT(K1)*AJ2*EO/SIGA)*(0.00464)
      PHIXB(KZ)=(DFNT(K1)*EO/SIGA)*(-0.00464)
    3 CONTINUE
    1 CONTINUE
   10 FORMAT(6E12.5)
      RETURN
      END
```

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13. ABSTRACT

An existing computer program for the geometrically non-linear analysis of arbitrarily loaded shells of revolution was modified to incorporate initial geometric imperfections into the buckling solution of an axially loaded circular cylindrical shell. Several different boundary conditions were considered, and the predicted buckling loads were examined to determine the significance of the initial imperfections and the boundary conditions for the shell stability problem. The computed buckling loads were compared to those found experimentally by Arbocz and Babcock at the California Institute of Technology. The computed results were generally higher than the experimental results. It was indicated that imperfection sensitivity is dependent on the boundary conditions of the cylinder.

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A computer study of the buckling of an imperfect cylindrical shell.